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Simple Mathematical Models of Mode Splitting of Hermetic Compressor Shells That Deviate from Axisymmetry

by

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ABSTRACT

A hermetic compressor shell is viewed as a circular cylinder with attached mass or stiffness and as an oval cylinder. The phenomenon that two similar modes of equal number of node lines exist at two different natural frequencies is discussed and controlling influences are pointed out.

INTRODUCTION

Knowledge of natural frequencies and modes of compressor shells has gained in importance because noise control of hermetic compressors has become more important. Natural frequencies and modes are valuable information by themselves and also in the frame work of modal analysis with a view of predicting the transmitted and radiated sound. More and more are they determined experimentally in an almost routine manner, and steps have been taken to obtain them theoretically. One phenomenon that has puzzled investigators occasionally is that often two very similar mode shapes (same number of node lines) exist at natural frequencies, that are so close together that the mode shapes are often mistakenly classified as the same mode. The purpose of this paper is to point out that they are separate modes and have to be recorded in order to obtain a full mode spectrum that is useful for modal analysis.

While the basic natural frequency solution of the circular cylindrical shell that is used as example was first worked out by Nowacki [1], the mode splitting solutions in their simple forms were worked out by the author using approaches that will be summarized in reference [2]. Obviously, a finite element program that does not use axisymmetry as simplification will also furnish mode splitting solutions if its root search routine is done properly, except that it will not explain what is going on.

PERFECTLY AXISYMMETRIC SHELL

This case is never realized since the necessary electric connections, suction and discharge tube penetrations and support arrangement will prevent it from occurring, even if the shell itself is perfectly round. Still, it is useful to discuss this case since it shows the phenomenon of non preferential direction of axisymmetric modes. Taking the idealized case of a perfect circular cylindrical shell, the experimenter notices that the wisdom that he has absorbed in an ordinary introductory vibration course is not any longer applicable. He has learned, on the example of a vibrating beam, most likely, that no matter where the location of the exciter is, the mode shape is invariant. Yet, in the case of the perfectly axisymmetric shell, he notices that the mode shape will orient itself such that one of its antinodes is always lined up with the exciter. See Fig. 1. The physical reason is, obviously, the axisymmetry, since there should obviously be no preferential direction. In addition, what he may not realize is that he has recorded an incomplete modal set if he records only one mode at that particular frequency.

To augment and clarify this discussion with some simple mathematics, we approximate the compressor shell as a simply supported circular cylinder and solve for the natural frequencies using the Donnell-Mushtari-Vlasov equations. We obtain [1]

$$\omega_{mn}^2 = \frac{1}{a^2} \left\{ \frac{(m\pi a/L)^2}{[(m\pi a/L)^2 + n^2]^2} + \frac{(h/a)^2}{12(1-\mu^2)} \right\} \left(\frac{E}{\rho} \right) \quad (1)$$

Where

a = radius [mm]
 L = length of shell [mm]
 h = thickness of shell [mm]
 μ = Poisson's ratio
 E = Young's modulus [N/mm²]
 ρ = mass density [Ns²/mm]

The integers $n = 0, 1, 2, \dots$ and $m = 1, 2, 3, \dots$ are identified by the modeshape of which we have to record two orthogonal sets

$$U_{3mn1}(x, \theta) = \sin(m\pi x/L) \cos n\theta \quad (2)$$

$$U_{3mn2}(x, \theta) = \sin(m\pi x/L) \sin n\theta \quad (3)$$

Chances are that the experimenter has only measured the first set. The node line pattern of both sets is the same, except that those of the second set are shifted by $\pi/2$. In the author's terminology, this case is a limiting case of node splitting since both sets occur at the same natural frequency. See Fig. 2 for the basic mode set for $n = 2$.

MODE SPLITTING CAUSED BY A MASS OR STIFFNESS

Let us now take the same shell, but consider an attached small point mass that disturbs the axisymmetry. The result is that the modes do now have a preferential direction. Their approximate theoretical expressions are still given by Eqs. (2) and (3), but their line-up is now such that in the set described by Eq. (2) the antinode is located at the location of mass while the set described by Eq. (3) will always have a node at the mass location. See Fig. 3. That is, the origin of the θ coordinate is at the mass. The set described by Eq. (3) occurs still at the natural frequencies given by Eq. (1), namely

$$\omega_{mn1}^2 = \omega_{mn}^2 \quad (4)$$

The set described by Eq. (2) occurs now at a natural frequency (worked out by the author):

$$\omega_{mn2}^2 = \omega_{mn}^2 (1 - \epsilon_{mn}^2) \quad (5)$$

where, approximately,

$$\epsilon_{mn}^2 = \frac{4M}{M_s} \sin^2(m\pi x^*/L) \quad (6)$$

and where

M = attached mass [Ns²/mm]
 M_s = total shell mass [Ns²/mm]
 x^* = location of attached mass [mm]
 L = length of shell [mm]

As it can be seen, for small antisymmetries, ϵ_{mn} is a small number and the two natural frequencies for modes of similar appearance are only separated by an amount $\epsilon_{mn} \omega_{mn}$.

In the case of a stiffness added at a single location, the result is similar, except that we have now (see Fig. 4):

$$\omega_{mn1}^2 = \omega_{mn}^2 \quad (7)$$

$$\omega_{mn2}^2 = \omega_{mn}^2 (1 + k_{mn}^2) \quad (8)$$

where

$$k_{mn}^2 = \frac{4K}{M\omega_{mn}^2} \sin^2(m\pi x^*/L) \quad (9)$$

and where K = attached stiffness [N/mm]. The only difference between the two cases is that the stiffness addition splits the modes in a way that one of the frequencies is larger than the original frequency, while the mass addition splits it in a way that one of the frequencies is smaller than the original one. It is easy to see that a combined addition of mass and stiffness will produce frequency pairs where one is lower and the other one is higher than the original frequency. If the added mass and stiffness have a natural frequency by themselves that coincides with one of the natural frequencies of the original shell, the split is roughly equal in both directions. The similarity to the dynamic damper application is apparent.

The effect of the non-axisymmetric support arrangement and compressor suspension can be understood in the same way, except that variations of the discussed behaviour may exist that should perhaps be investigated in a similarly simple fashion.

MODE SPLITTING CAUSED BY OVALNESS

Interpreting an oval shell as a shell that is a deviation from an axisymmetric shell by defining a curvature in circumferential direction $1/R_\theta$ as a deviation from the equivalent curvature $1/a$ of a round shell

$$\frac{1}{R_\theta} = \frac{1}{a} [1 + \epsilon(\theta)] \quad (10)$$

where

a = radius of equivalent circular cylindrical shell [mm] = $C/L\pi$

C = circumference of oval shell [mm]

$\varepsilon(\theta)$ = a function periodic by 2π

R_0 = actual radius of curvature [mm]

one can show that we again obtain two different sets of natural modes that look similar when one counts the number of node lines. The natural frequencies are again split into two values (caused by the ovalness) and are approximately given by

$$\omega_{mn1}^2 = \omega_{mn}^2 + A_{mn} \int_0^{2\pi} \varepsilon(\theta) \cos^2 n\theta d\theta \quad (11)$$

$$\omega_{mn2}^2 = \omega_{mn}^2 + A_{mn} \int_0^{2\pi} \varepsilon(\theta) \sin^2 n\theta d\theta \quad (12)$$

where

$$A_{mn} = \frac{E}{\rho a^2} \frac{(m\pi/L)^2}{[(n/a)^2 + (m\pi/L)^2]^2}$$

Typically one would expect the integral of Eq. (12) to have a negative value. The basic mode set for $n = 2$ is shown in Fig. 5.

CONCLUSION AND DISCUSSION

This paper shows that the often observed phenomenon of mode splitting of compressor shells can be explained by relatively simple models. It points out that it is not permissible for the experimenter to lump split modes into one and not report them separately. How precisely such an omission affects the result of a modal analysis is beyond the scope of this paper. One effect is that beating may occur under certain conditions because of split modes.

The paper shows that any kind of deviation from axisymmetry causes mode splitting.

REFERENCES

- [1] W. Nowacki, 1963, "Dynamics of Elastic Systems", New York, John Wiley.
- [2] W. Soedel, "Vibration of Shells and Plates", to be published, Marcel Dekker.

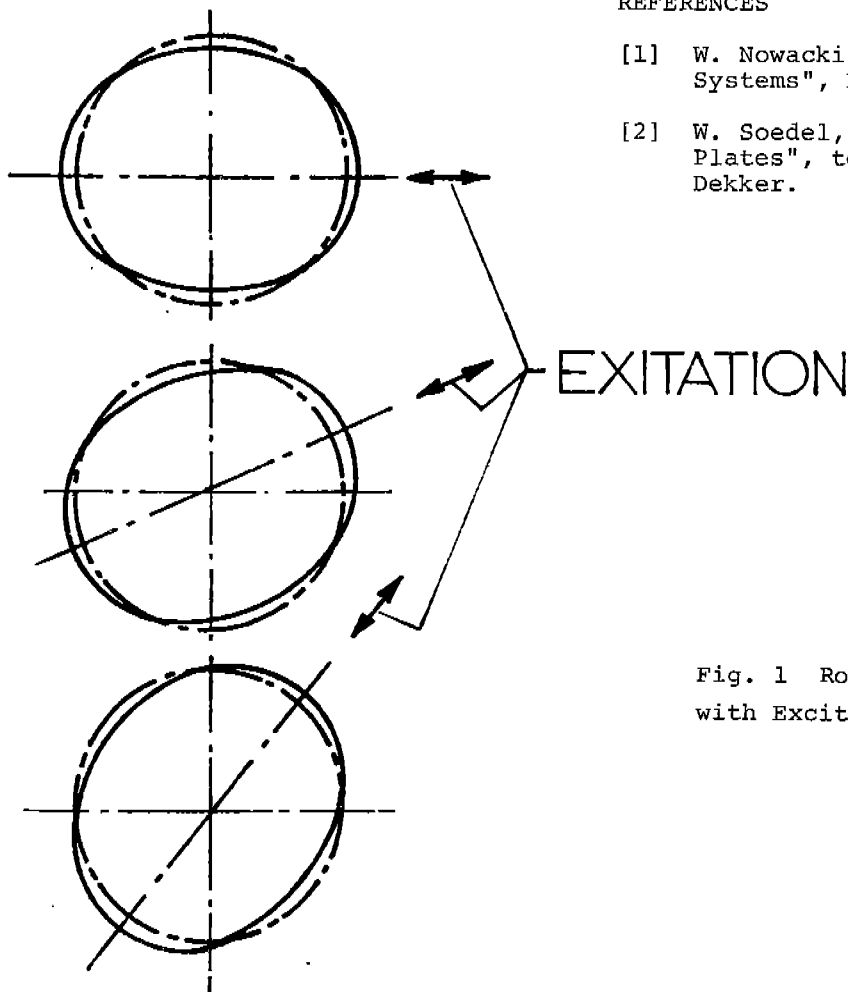


Fig. 1 Rotation of Excited Mode with Exciter Location

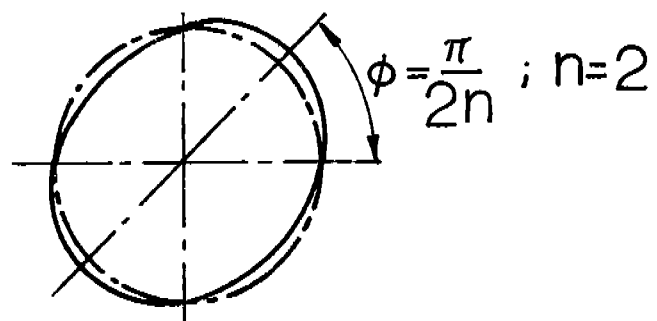
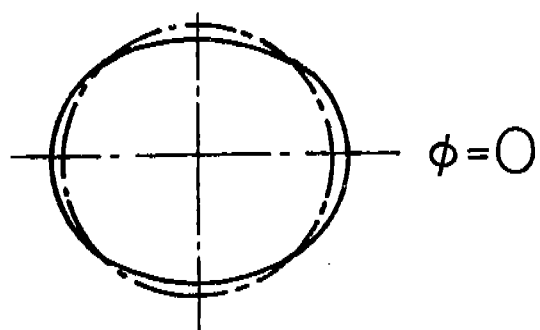


Fig. 2 Base Modes for $n = 2$

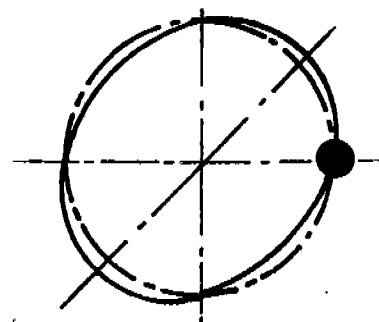
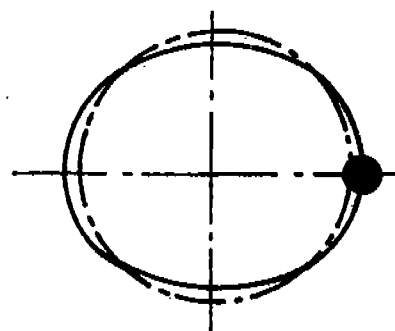


Fig. 3 Split Modes for $n = 2$
when Mass is Attached to Shell

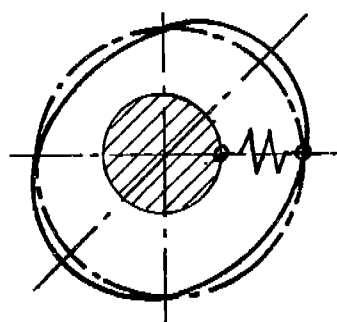
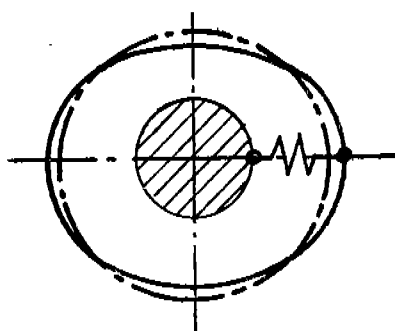


Fig. 4 Split Modes for $n = 2$
when Stiffness is Attached to Shell

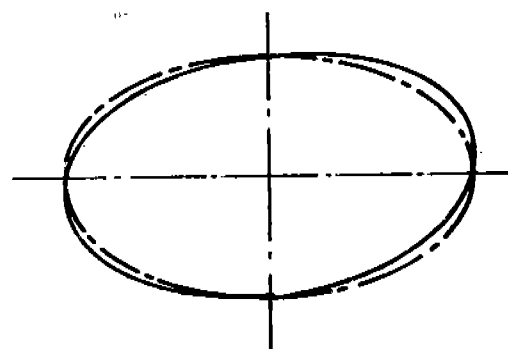
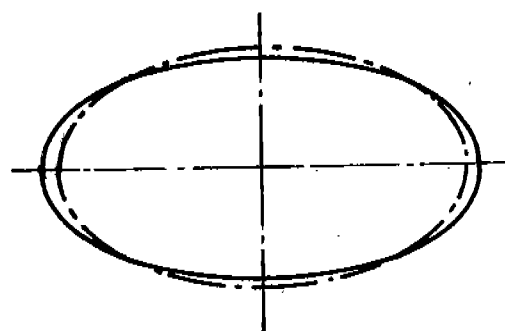


Fig. 5 Split Modes for $n = 2$
When Shell is Oval